CHAPTER 13

DEFORMATION DURING SOLIDIFICATION AND HOT TEARING

Movies from experiments and simulations of hot tearing can be found at: http://solidification.org/Movies/tearing.html

13.1 INTRODUCTION

We saw in the previous chapter that porosity can be caused by the inability to feed solidification shrinkage due to pressure losses through the mushy zone (shrinkage porosity), or by the segregation of too much dissolved gas in the liquid (gas porosity). Although the same equations are used to describe the two cases, they differ in both the mechanism and possible preventive measures for pore formation. Gas porosity can be eliminated by an appropriate melt treatment, such as degassing, which removes excess dissolved gas prior to casting, or by preventing reactions between the melt and oxygen or water which lead to excess gas in the melt. However, the alloy’s tendency to form shrinkage porosity is a more intrinsic characteristic, caused by the formation of a dense phase during late-stage solidification. Consequently, it is harder to avoid.

One way to eliminate shrinkage porosity is to compress the mushy zone during freezing. Indeed, we show in Chap. 14 that the partially coherent solid of a mushy zone can be compacted or expanded like a sponge. Putting this concept in the context of Eq. (12.5), in regions of a casting that are under compression, i.e., where $\nabla \cdot v_s < 0$, the deformation of the solid can counterbalance the density increase and thus maintain $g_p = 0$, even when there is no feeding, i.e., for $v_t = 0$. This is the idea behind the squeeze casting process, in which an extra compressive stress is applied at certain locations at prescribed times.
Deformation during solidification and hot tearing

Fig. 13.1 Hot cracks observed (a) at the center of a DC cast and (b) at the weld centerline of an aluminum alloy.

However, in many solidification processes such as shape casting, continuous casting or welding, some locations are subjected to tension rather than to compression during freezing. In this case, the deformation of the solid skeleton adds to the solidification shrinkage, thereby inducing even more liquid suction in the mushy zone (see Fig. 12.5).

Key Concept 13.1: Hot tearing definition

A lack of feeding of mushy regions under tension (or shear) will result in what is called **hot tearing** or **hot cracking**, a defect that is similar to shrinkage porosity, but that also requires tensile stresses. Whereas microporosity is fairly uniformly distributed within the mushy zone, hot tears are localized at grain boundaries.

Figure 13.1(a) and (b) show, respectively, hot tears formed at the center of a DC-cast Al billet, and along the weld centerline of an aluminum sheet. During both processes, the tears occur in regions where the thermal strains are the largest.

Feeding has already been treated in the chapter on porosity. Therefore, the present chapter focuses on how and where tensile deformation arises during solidification. We show in Sect. 13.2 that non-uniform cooling of the solid induces non-uniform thermal contractions, deformations and stresses. Near the solidus temperature, the elastic limit of the solid is low and viscoplastic deformation of the solid normally occurs at constant volume, i.e., with $\nabla \cdot v_s = 0$. We demonstrate in Sect. 13.3 that a two-phase region with a certain amount of remaining liquid has a much more complicated constitutive response, and includes the possibility of viscoplastic deformation where $\nabla \cdot v_s \neq 0$. 
The last two sections of the chapter are dedicated to characteristics, observations and measurements of hot tearing (Sect. 13.4), and to hot cracking models (Sect. 13.5).

13.2 THERMOMECHANICS OF CASTINGS

13.2.1 Origins of thermal stresses

In this section, we explore, through the use of simple models, the origin of deformation and stress during solidification. Dimensional changes are required to accommodate the volume change upon solidification, and more importantly, the thermal contraction of the solid upon cooling to room temperature. These dimensional changes produce stresses if one or more of the following conditions are met: non-uniform temperature, non-uniform mechanical properties, constraints induced by the mold. We illustrate this concept now, using a simple 1-D model of a thin, hot coating at temperature $T_{\text{sol}}$ attached to a cold substrate at temperature $T_0$, and examine the stress state of the assembled pair when equilibrium is established at $T_0$. One might encounter such a situation in plasma spraying of a thin (and hot) coating over a component.

In order to keep the analysis simple, we consider both the coating and the substrate to be thin enough that they are both in a state of plane stress. We first address the case where the substrate is constrained at its ends, as illustrated in Fig. 13.2 (top). The assembly is assumed to be free to deform in the direction perpendicular to the page, and thus the only non-zero stress is $\sigma_{xx}$. It is conceptually useful to separate the process into two steps. In the first step, we imagine that the coating is detached from the substrate and cooled to $T_0$ without constraint. This would be accompanied by thermal shrinkage $\Delta L_x$, and associated thermal strain $\varepsilon_{xx}^{th}$.

![Fig. 13.2 A simple block model of thermal contraction for the case of a thin hot solid cooling over a (top) large undeformable and (bottom) deformable substrate.](image-url)
Key Concept 13.2: Thermal strains

In 1-D, when a solid component of initial length $L_x$ has a temperature that changes from $T_{initial}$ to $T_{final}$ and is free to deform, its thermal expansion (or contraction) is given by

$$\Delta L_x = L_x \alpha T_s (T_{final} - T_{initial}) \quad (13.1)$$

where $\alpha T_s = \beta T_s / 3$ is the linear thermal expansion coefficient of the material, assumed constant in this equation. The associated thermal strain $\varepsilon_{xx}^{th}$ is thus given by

$$\varepsilon_{xx}^{th} = \frac{\Delta L_x}{L_x} = \alpha T_s (T_{final} - T_{initial}) \quad (13.2)$$

In the present case, the thermal strain of the coating is given by

$$\varepsilon_{xx}^{th} = \frac{\Delta L_x}{L_x} = \alpha T_s (T_0 - T_{sol}) < 0 \quad (13.3)$$

In the second step, we re-attach the cold coating to the undeformed substrate, which requires the layer to be mechanically stretched until its length is once again $L_x$. The total strain of the coating is then zero, since its length remains unchanged from its original value, which implies that the mechanical strain balances the thermal strain,

$$\varepsilon_{xx} = \varepsilon_{xx}^{th} + \varepsilon_{xx}^{el} + \varepsilon_{xx}^{vp} = 0 \quad (13.4)$$

where the superscripts “el” and “vp” refer to elastic and viscoplastic components of the mechanical strain, respectively.

If the deformation is entirely elastic, then $\varepsilon_{xx}^{vp} = 0$ and the elastic stress is obtained using Eq. (13.2) and Hooke’s law

$$\sigma_{xx} = E \alpha T_s (T_{sol} - T_0) > 0 \quad (13.5)$$

where $E$ is the elastic modulus of the coating. The case of an elastic-viscoplastic solid is treated in Exercise 13.1.

Let us now consider the case where the substrate is also deformable, and bending is permitted in the $x$-$z$ plane, as illustrated in Fig. 13.2 (bottom). In this case, when the cooled coating is re-attached to the substrate, the coating remains under tension at the interface, whereas the substrate is under compression. This causes bending of the pair. To provide a more quantitative example, suppose that the coating and the substrate each have the same thickness, $e$, the same elastic modulus, $E$, and that the deformation is entirely elastic. The length of any chord, $L'_x(z)$, is shown in Ex. 13.2 to be

$$L'_x(z) = L_x \left[ 1 - \alpha T_s (T_{sol} - T_0) \left( \frac{1}{2} + \frac{3z}{4e} \right) \right] - e \leq z \leq e \quad (13.6)$$
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Fig. 13.3 Stress in a coating-substrate of equal thickness and elastic modulus as a function of the normalized thickness, \( z/e \). The stress has been normalized by the factor \( E\alpha_T s(T_{sol} - T_0) \).

where \( z \) is the distance measured from the centerline. The chord length at the interface position (\( z = 0 \)) is reduced by an amount equal to half of the free thermal contraction of the coating, and the radius of curvature at the interface, \( R \), is given by

\[
\frac{1}{R} = -\frac{1}{L_x} \frac{dL_x'(z)}{dz} \Rightarrow R = \frac{4e}{3\alpha_T s(T_{sol} - T_0)} \quad (13.7)
\]

Using a typical value for aluminum, \( \alpha_T s = 2 \times 10^{-5} \text{ K}^{-1} \), and a temperature difference of 500 K, the radius of curvature \( R = 133 e \). The stress in the substrate is given by

\[
\sigma_{sub}^{xx} = E \frac{L_x'(z) - L_x}{L_x} = -E\alpha_T s(T_{sol} - T_0) \left( \frac{1}{2} + \frac{3z}{4e} \right) - e \leq z \leq 0 \quad (13.8)
\]

The length for the free coating at \( T_0 \) is not \( L_x \), but rather \( L_x(1 - \alpha_T s(T_{sol} - T_0)) \), and the stress in the coating therefore becomes

\[
\sigma_{coat}^{xx} = E \frac{L_x'(z) - L_x(1 - \alpha_T s(T_{sol} - T_0))}{L_x} = E\alpha_T s(T_{sol} - T_0) \left( \frac{1}{2} - \frac{3z}{4e} \right) 0 \leq z \leq e \quad (13.9)
\]

Thus, we see that the stresses in the coating and substrate at the interface are tensile and compressive, respectively, but as one moves towards the free surfaces, the sign of the stress changes in both entities (see Fig. 13.3).

Although simple, this “block model” serves to illustrate two important features of thermal deformation: (i) A component that is not constrained
by external forces, e.g., by a rigid mold or welding clamps, bends towards
the hottest region (Fig. 13.2); (ii) The interfacial region at the interior of
the curvature tends to be under tension, whereas the external part is sub-
jected to compression. Such simple models are very helpful to better un-
derstand more complex solidification processes, such as those presented in
Sect. 13.2.3.

13.2.2 General formalism for a fully solid material

In Sect. 4.3.2, we demonstrated that the total mechanical strain $\varepsilon_{ij}$ can be
decomposed into elastic and viscoplastic contributions, $\varepsilon_{ij}^{el}$ and $\varepsilon_{ij}^{vp}$, respectively. In order to model thermal stresses, as in the simple thermoelastic
block model described in the previous section, we also include the ther-
mal strain, $\varepsilon_{ij}^{th}$, induced by thermal contractions. In certain materials, one
must also consider an additional strain contribution, $\varepsilon_{ij}^{tr}$, induced by solid
state transformations, e.g., the volume change of approximately 4% asso-
ciated with the austenite-martensite transformation in steels. These four
contributions add up to give the total strain.

Key Concept 13.3: Total strain of a solid

The total strain $\varepsilon_{ij}$ of a solid that undergoes temperature changes as
well as solid state transformations is defined by additive decomposi-
tion of the strains due to the various phenomena:

$$\varepsilon_{ij} = \varepsilon_{ij}^{el} + \varepsilon_{ij}^{vp} + \varepsilon_{ij}^{th} + \varepsilon_{ij}^{tr}$$ (13.10)

where $\varepsilon_{ij}^{el}$, $\varepsilon_{ij}^{vp}$, $\varepsilon_{ij}^{th}$ and $\varepsilon_{ij}^{tr}$ correspond to the elastic, viscoplastic, ther-
mal and phase transformational components of the strain tensor, re-
spectively. If the material is isotropic, the thermal and transformation
strain tensors each reduce to a single scalar material parameter, such that

$$\varepsilon_{ij}^{th}(x, t) = \alpha T_s(T(x, t) - T_{ref})\delta_{ij}; \quad \varepsilon_{ij}^{tr}(x, t) = \frac{\beta_{tr}}{3} g_{tr}(x, t)\delta_{ij}$$ (13.11)

where $T_{ref}$ is a reference temperature at which the strain is defined
to be zero, $\alpha T_s$ is the linear thermal expansion coefficient, $g_{tr}$ is the
volume fraction of solid that has transformed into the new phase, and
$\beta_{tr}$ is the volumetric expansion coefficient associated with the solid-
state transformation.

Notice that in one dimension, $\varepsilon_{xx}^{th}$ reduces to Eq. (13.2). In general,
when computing thermal stresses, the temperature field is obtained first
in a separate calculation. Computation of $g_{tr}$ requires an additional micro-
scopic model of transformation. In most cases, such models are obtained
from Time-Temperature-Transformation (TTT) diagrams (cf. Jacot et al.
Since the thermal and transformational strains depend only on temperature (and possibly on the cooling rate), they become “loads” in the momentum balance equations that are subsequently solved for the displacement field, $u_i(x, t)$. Appropriate constitutive models are supplied in order to relate the stress to the elastic and viscoplastic strains (see Chap. 4). Although these equations are generally nonlinear, many commercial codes are available for their solution.

In most solidification problems, it is not necessary to consider the heat generated by deformation in the energy equation. The thermal and stress-deformation fields are nevertheless coupled in several ways. One form of coupling derives from the temperature dependence of the mechanical and physical properties of the material. The shape of the isotherms, through the magnitude and orientation of the thermal gradient, affects the deformation and stresses, as we saw in the simple block model. Deformation can also affect the thermal field by altering the boundary conditions. For example, air gap formation decreases the heat transfer coefficient between the mold and casting by as much as an order of magnitude. On the other hand, increased contact pressure in other regions can raise the local heat transfer coefficient by an almost as large a factor.

### 13.2.3 Examples

Hot tearing is a major problem in continuous casting processes as well as in welding, as illustrated in Fig. 13.1. Figure 13.4 demonstrates the

![Fig. 13.4 A schematic of hot tear formation (a) in a DC casting of aluminum and (b) during e-beam or laser welding. The shapes of the transverse cross-sections before and after welding are shown at the bottom of the figure.](image-url)